

MR3010166 (Review) [53A10](#)

Morabito, Filippo (KR-KIAS-SM);

Rodríguez, M. Magdalena [**Rodríguez Pérez, M. Magdalena**] (E-GRAN-G)

Classification of rotational special Weingarten surfaces of minimal type in $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$. (English summary)

Math. Z. **273** (2013), *no. 1-2*, 379–399.1432-1823

In the paper under review, the special Weingarten surfaces that are studied are surfaces for which there exists a function $f \in \mathcal{C}^1([0, +\infty))$ such that $H = f(H^2 - K_e)$, where H and K_e denote respectively the mean curvature and the extrinsic curvature of the surface and f satisfies $4x(f'(x))^2 < 1$ for any $x \geq 0$. These special Weingarten surfaces are called of minimal type if $f(0) = 0$.

The authors prove the necessary lemmas and propositions in order to establish two classification theorems of rotational special Weingarten surfaces of minimal type in $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$. Summarized, these theorems state that:

- a complete rotational Weingarten surface of minimal type in $\mathbb{H}^2 \times \mathbb{R}$ is either a horizontal slice or a catenoidal type surface whose determining functions satisfy certain conditions;
- a complete rotational Weingarten surface of minimal type in $\mathbb{S}^2 \times \mathbb{R}$ is either a horizontal slice, or a vertical cylinder, or an unduloidal type surface whose determining functions satisfy certain conditions.

These results generalize several known theorems for minimal surfaces.

Also, the existence of non-complete examples of rotational special Weingarten surfaces of minimal type is shown.

Reviewed by [Wendy Goemans](#)

References

1. Chern, S.S.: On special W-surface. *Proc. Am. Math. Soc.* **6**, 783–786 (1955) [MR0074857 \(17,657h\)](#)
2. Elbert M.F.: Constant positive 2-mean curvature hypersurfaces. *Ill. J. Math.* **46**(1), 247–267 (2002) [MR1936088 \(2003g:53103\)](#)
3. Hartman, P., Wintner, W.: Umbilical points and W-surfaces. *Am. J. Math.* **76**, 502–508 (1954) [MR0063082 \(16,68a\)](#)
4. Hopf, H.: Differential geometry in the large. *Lecture Notes in Mathematics*, vol. 1000. Springer, Berlin (1983) [MR0707850 \(85b:53001\)](#)
5. Korevaar, N., Kusner, R., Solomon, B.: The structure of complete embedded surfaces with constant mean curvature. *J. Differ. Geom.* **30**, 465–503 (1989) [MR1010168 \(90g:53011\)](#)
6. Meeks, W.H. III.: The topology and geometry of embedded surfaces of constant mean curvature. *J. Differ. Geom.* **27**, 539–552 (1988) [MR0940118 \(89h:53025\)](#)
7. Nelli, B., Rosenberg, H.: Minimal surfaces in $\mathbb{H}^2 \times \mathbb{R}$, *Bull. Braz. Math. Soc.* **33**, 263–292.

MR1940353, Zbl 1038.53011 (2002) [MR1940353 \(2004d:53014\)](#)

8. Nelli, B., Sa Earp, R., Santos, W., Toubiana, E.: Uniqueness of H -surfaces in $\mathbb{H}^2 \times \mathbb{R}$ $|H| \leq 1/2$, with boundary one or two parallel horizontal circles. *Ann. Glob. Anal. Geom.* **33**, 307–321 (2008) [MR2395188 \(2009k:53017\)](#)
9. Pedrosa, R., Ritoré, M.: Isoperimetric domains in the Riemannian product of a circle with a simply connected space form and applications to free boundary value problems. *Indiana Univ. Math. J.* **48**, 1357–1394. MR1757077, Zbl 0956.53049 (1999) [MR1757077 \(2001k:53120\)](#)
10. Rosenberg, H.: Minimal surfaces in $\mathbb{M}^2 \times \mathbb{R}$. III. *J. Math.* **46**(4), 1177–1195 (2002) [MR1988257 \(2004d:53015\)](#)
11. Rosenberg, H., Sa Earp, R.: The geometry of embedded special surfaces in \mathbb{R}^3 ; e.g. satisfying $aH + bK = 1$, where a and b are positive. *Duke Math. J.* **73**, 123–148 (1994) [MR1173038 \(93k:53010\)](#)
12. Sa Earp, R., Toubiana, E.: Sur les surfaces spéciales de Weingarten de type minimal. *Bull. Soc. Mat. Bras.* **26**(2), 129–148 (1995) [MR1364263 \(96j:53006\)](#)
13. Sa Earp, R., Toubiana, E.: Classification des surfaces de type Delaunay. *Am. J. Math.* **121**, 671–700 (1999) [MR1738404 \(2001b:53004\)](#)
14. Sa Earp, R., Toubiana, E.: Symmetry of properly embedded special Weingarten surfaces in \mathbb{H}^3 . *Trans. Am. Math. Soc.* **351**(12), 4693–4711 (1999) [MR1675186 \(2000c:53007\)](#)
15. Schoen, R.: Uniqueness, symmetry, and embeddedness of minimal surfaces. *J. Differ. Geom.* **18**, 791–809 (1983) [MR0730928 \(85f:53011\)](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

© Copyright American Mathematical Society 2014